Complex Numbers 1990 AHSME #22 Solution #2

This second solution continues from where we have

$$w_i = 64^{\frac{1}{6}}\left(\cos\left(\frac{\pi+2k\pi}{n}\right)+i\sin\left(\frac{\pi+2k\pi}{n}\right)\right)$$

for $k = 0,1,2,3,4,$ and $5.$

For the first solution, we simply use values of k to find the complex solutions, however, there's a more intuitive way which I personally prefer. Let us first figure out the first solution.

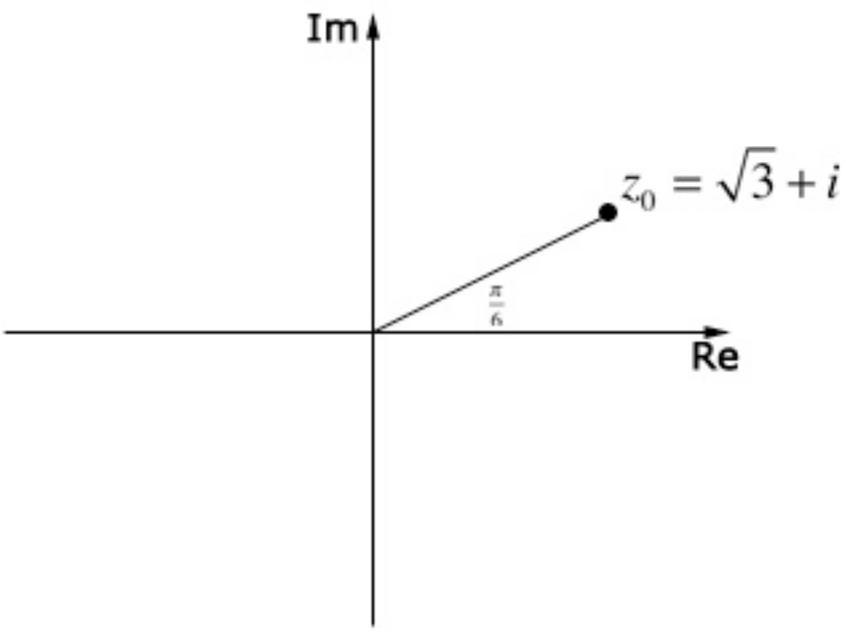
$$z_0 = \sqrt{3}+i$$

What we are going to do from here is to draw this number on the complex plane then use its geometric properties to figure out the other solutions. The magnitude and argument can be easily calculated like so.

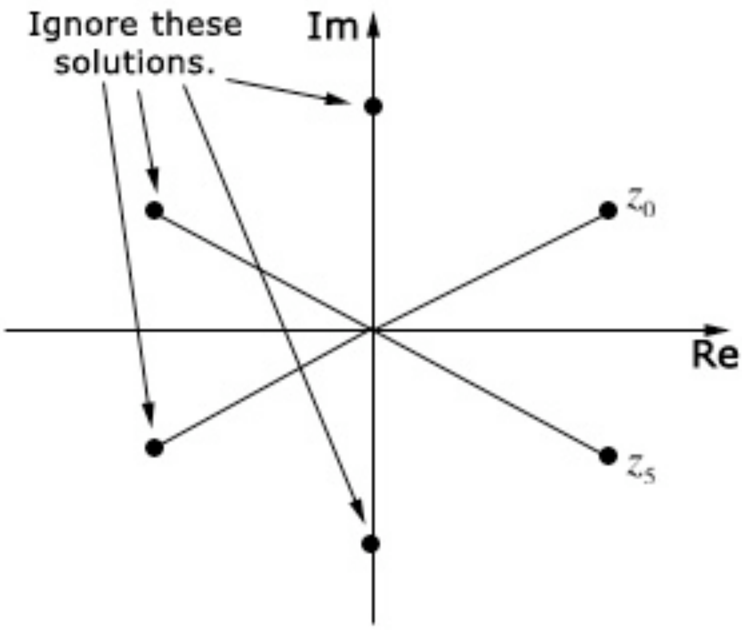
$$\left\|z_0\right\| = \sqrt{((\sqrt{3})^2+(1)^2)}$$

$$\arg(z_0) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

which gives us the complex graph,



Given the property that the nth roots will lie on a circle with the angles between each of them as 2 /6, we can now draw the other complex numbers and in doing so, we simply ignore those on the real axis with a less than or equal to 0, that is on the left side of the real axis.



This eliminates the work involve, adding to the fact that it's a smarter way to find the solution, such that we simply use the values of k=0 and k=5 for the solutions of z. Furthermore, adding to the fact that they are conjugates of each other, we have

$$z_0 \cdot z_5 = \|z_0\|^2 = \left(\sqrt{3}\right)^2 + \left(1\right)^2 = 4$$

A more elegant solution I should add.