Complex Numbers

Conjugate and Magnitude

The **complex conjugate** of a complex number $z = a + bi$, is given by

$$\overline{z} = a - bi$$

Take note that if *z* has a real number, i.e. to say that *b*=0, then we also have $z = \overline{z}$, since the imaginary part is 0 and so the negative of the imaginary part is also 0.

In interest of time, we shall show the following algebraic properties of the complex conjugate without proof. They are,

$$\overline{\overline{z}} = z,\ \overline{z+w} = \overline{z} + \overline{w},\ \overline{z \cdot w} = \overline{z} \cdot \overline{w}, \text{and} \overline{\left(\frac{1}{z}\right)} = \frac{1}{\overline{z}}$$

of which we will rarely use the last result.

NOTE: In the video, I did mention that the conjugate is like an 'inverse'. This is certainly not true. Because $z \cdot \overline{z}$ does not give us the identity, assuming an identity of the complex plane exists. What I meant to say is that there is a conjugate for every complex number.

Moving on, we define the **magnitude**, or sometimes known as the *absolute value* of the complex number $z = a + bi$ by

$$|z| = \sqrt{a^2 + b^2} = \sqrt{z \cdot \overline{z}}$$

Just as the name suggest, the magnitude gives a scalar value. We shall later see the geometric interpretation of this quantity. However at this point, it is vital that we recognize that in most cases, $|z|^2 \neq z^2$. This is simply shown that when we take the square of a complex number, we must do the standard multiplying procedures with complex numbers. $|z|^2$ means that we find the magnitude first and then square it giving us a real value. The exception to the case is when *z* has no imaginary part.

We again have the following result.

$$|\overline{z}| = |a - bi| = \sqrt{a^2 + b^2} = |z|$$

To conclude the section, we shall talk about a technique we use in the complex plane which is similar to the technique in real-numbers algebra called *rationalizing the denominator*.

To recall, to rationalizing a denominator such as $\left(1+\sqrt{2}\right)$, we multiply by the number 1 expressed as $\frac{\left(1-\sqrt{2}\right)}{\left(1-\sqrt{2}\right)}$. This way we can eliminate the root 2. In complex numbers, rationalizing the denominator involves removing the imaginary part in the denominator using a parallel method. We generalize the case by using any complex number *z*.

$$\begin{aligned}\frac{1}{z} = \frac{1}{a+bi} &= \frac{1}{a+bi}\cdot\frac{a-bi}{a-bi}\\ &= \frac{a-bi}{a^2+b^2}\\ &= \frac{\overline{z}}{|z|^2}\end{aligned}$$

Which gives us the fairly useful result.

$$\frac{1}{z} = \frac{\overline{z}}{|z|^2}$$