Differential Vector Calculus

Acceleration Components

Previously, I showed you a way in finding the acceleration of a particle which is simply the first derivative of the velocity vector. Now, with our newly defined terms, κ the curvature and ρ the radius of curvature, along with the unit normal and unit tangent vector, we shall write the acceleration vector in another form.

Since **T** is the unit tangent vector and **N** is the unit normal vector, we can now write the acceleration vector as such.

$$\vec{\mathbf{a}} = \frac{dv}{dt}\vec{\mathbf{T}} + \frac{1}{\rho}v^2\vec{\mathbf{N}}$$

We shall now attempt to proof the above. You need to have a good grasp of differentiating products of real-value functions and vector functions.

First, notice that in terms of a parameter *t,* we have

$$\vec{\mathbf{T}}(t) = \frac{1}{\left\|\vec{\mathbf{F}}'(t)\right\|}\vec{\mathbf{F}}'(t) = \frac{1}{v}\vec{\mathbf{v}}$$

which follows to

$$\vec{\mathbf{v}} = v\vec{\mathbf{T}}$$

Then

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{d}{dt}\left[v\vec{\mathbf{T}}\right] = \frac{dv}{dt}\vec{\mathbf{T}} + v\frac{d\vec{\mathbf{T}}}{dt}, \text{ via product rule}$$

$$= v\frac{d\vec{\mathbf{T}}}{dt} + v\frac{ds}{dt}\frac{d\vec{\mathbf{T}}}{ds} \text{ via chain rule}$$

$$= v\frac{d\vec{\mathbf{T}}}{dt} + v^2\frac{d\vec{\mathbf{T}}}{ds}$$

since $v = \frac{ds}{dt}$.

But from the previous section, we defined the unit normal vector as $\vec{\mathbf{N}} = \rho\left(\frac{d\vec{\mathbf{T}}}{ds}\right)$, and so $\frac{d\vec{\mathbf{T}}}{ds} = \frac{1}{\rho}\vec{\mathbf{N}}$, giving us

$$\vec{\mathbf{a}} = \frac{dv}{dt}\vec{\mathbf{T}} + \frac{1}{\rho}v^2\vec{\mathbf{N}}$$

Looking at this equation carefully, notice that the tangential and normal **vectors** are both of unit length, meaning to say that the magnitude of these vectors is dictated by the scalar functions before them.

Thinking along these lines, the scalar $\frac{dv}{dt}$ is called the *tangential component* of the acceleration and is denoted by a_T. The scalar $\left(\frac{1}{\rho}\right)v^2$ is called the *normal component* of the acceleration and is denoted by a_N. With these definitions, another way to write the acceleration vector is

$$\vec{\mathbf{a}} = a_T\vec{\mathbf{T}} + a_N\vec{\mathbf{N}}$$

I would like to kindly remind the reader that this is *another* way of writing the acceleration vector instead of take the first derivative of the velocity vector. As you shall see in the later section, which way the reader finds the acceleration vector should be based on the convenience. Usually when the situation starts out with the position vector in terms of *t,* then taking the derivatives is desirable. If the position vector is in terms of the arc length, then this method should be more suitable.