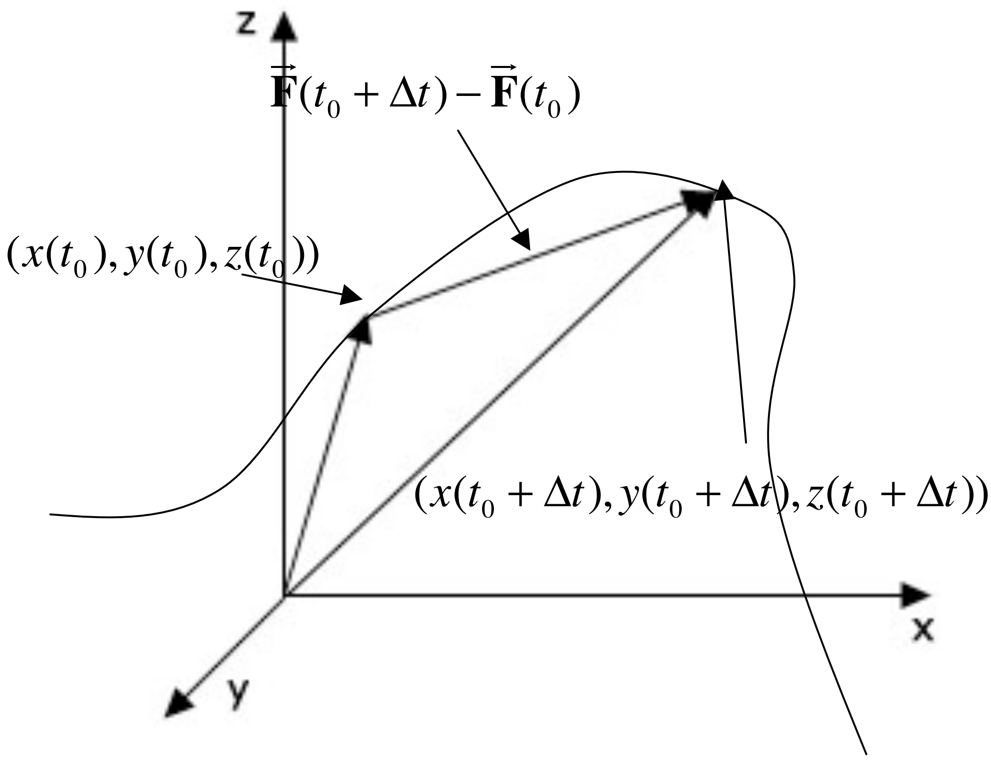
Differential Vector Calculus
The first derivative

We all know that the first derivative is the tangent to the curve at a given point on the curve. Using a similar argument, we are incline to think that the first derivative $\vec{\mathbf{F}}(t)$ is the tangent vector to a given curve defined by the vector function. To understand this, we will look at the 'first principles' of this derivative.

Consider, from the parallelogram law, the vector

$$\vec{\mathbf{F}}(t_0 + \Delta t) - \vec{\mathbf{F}}(t_0)$$

which is represented by the arrow from the point $(x(t_0), y(t_0), z(t_0))$ to the point $(x(t_0 + \Delta t), y(t_0 + \Delta t), z(t_0 + \Delta t))$ as shown in the diagram.



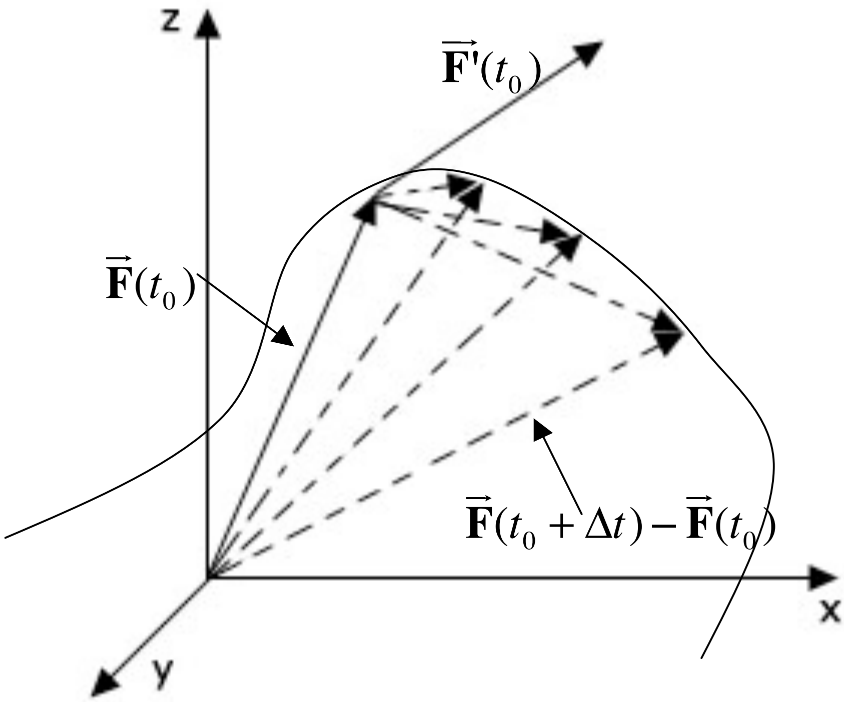
Since Δt is a nonzero scalar, the vector

$$\frac{1}{\Delta t}\left[\vec{\mathbf{F}}(t_0 + \Delta t) - \vec{\mathbf{F}}(t_0)\right]$$

is also along the line from $(x(t_0+\Delta t), y(t_0+\Delta t), z(t_0+\Delta t))$ to $(x(t_0+\Delta t), y(t_0+\Delta t), z(t_0+\Delta t))$. In terms of components,

$$\frac{1}{\Delta t}\left[\vec{\mathbf{F}}(t_0+\Delta t)-\vec{\mathbf{F}}(t_0)\right] = \frac{x(t_0+\Delta t)-x(t_0)}{\Delta t}\mathbf{i}+$$
$$= \frac{y(t_0+\Delta t)-y(t_0)}{\Delta t}\mathbf{j}+$$
$$= \frac{z(t_0+\Delta t)-z(t_0)}{\Delta t}\mathbf{k}$$

When we take the limit $\Delta t \to 0$, the left hand side of the above equation moves into a position tangent to the curve at $(x(t_0), y(t_0), z(t_0))$, while the right hand side of the equation approaches $x'(t_0)\mathbf{i}+y'(t_0)\mathbf{j}+z'(t_0)\mathbf{k}$ as pictured below.



This justifies our thinking that $\vec{\mathbf{F}}'(t_0)$ is the tangent to the curve at $(x(t_0), y(t_0), z(t_0))$.

And so, when we are given the curve with the equations $x = x(t), y = y(t), z = z(t)$ and ask for the tangent to that curve at the point t_0, we mean the vector $\vec{\mathbf{F}}'(t_0)$ where $\vec{\mathbf{F}}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ is the position vector of that curve.

Lastly, in calculating $\vec{\mathbf{F}}'(t_0)$, we will get a vector which, though we are not explicitly told where it starts from, should be draw at the point $(x(t_0), y(t_0), z(t_0))$ as it is after all a tangent vector. Just like how we would draw the tangent at a point of a curve in the x-y plane.