Differential Vector Calculus
Introduction

So you think you are ready to tackle calculus in the realm of 3-dimensional vectors? If that's you, then I welcome you to the vector calculus course where we'll start with differentiation.

Before you dive into this chapter, I believe that you should be equiped with a sound knowledge of the algebra and geometry of vectors in the plane, in three-space, and subsequently in a $\mathbb{R}^4$ plane. You will undoubtedly use such algebraic properties in manipulating and even proving most of the results. The reader should be able to quickly recall basic definitions like the dot and cross product of 2 vectors.

For most parts of this chapter and the soon to be release integral vector calculus, we will combine vector algebra and geometry with the processes of limits and continuity to develop new ideas of how the calculus is used in such areas.

Since interpreting the calculus in 3 dimensions can sometimes be a departure from its 2 dimensions roots, we will use that little things called 'intuition' to shed some meaning in the definitions. While I will always attempt to prove results conclusively with a mixture of algebra and geometry, more often than not it is intuition that gives the student that initial step in understanding what's going on.

Before we proceed, I would like to sum up by explaining the difference between of one variable and several variable calculus which the reader may or may not be familiar with.

Let's look at the position vector presented before us.

$$\vec{\mathbf{F}}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$$

While we did use the functions $x(t), y(t), z(t)$ to represent the vector, please know that this is *not* a vector function of several variables. The three component functions $x(t), y(t), z(t)$ are all in terms of one and the same variable, that is *t*.

Now let's look at another function, not necessarily a vector.

$$f(x, y) = \frac{xy}{x^2 - y}$$

The above function is based on several variables, in this case, it is 2. Note that the variables *x,* and *y,* are independent of each other meaning to say we can pick a value for *x* and a value of *y* where there is no relationship between the 2, hence the term multivariable. The function requires that we pick a value for each *x and y*.

Lastly, I would like to emphasize that there is no difference between the vectors written as $\mathbf{F}(t)$ **and** $\vec{\mathbf{F}}(t)$. The arrow drawn above the vector is simply an easier way to represent the vector compared to bolding it when calculations are done on paper. At times, the two different ways of writing the vector may be interchangeable. To that, I apologies for the lack of consistency. Nonetheless, they mean the same thing.