Differential Vector Calculus

## Vector Function of one variable

In most parts of this chapter, we will be dealing with vector functions of one variable. It is vital that the student gets acquainted which such functions as a sound knowledge is required when dealing with vector functions of several variables in the vector integral calculus A vector function takes the form:

$$\vec{\mathbf{F}}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$$

Such a function is referred to as a *vector function of one real variable* where the variable concern is *t*. It takes a scalar real value and is sometimes termed parameter. The vector $\vec{\mathbf{F}}(t)$ is of course a vector, having both direction and magnitude. For example, we might have,

$$\vec{\mathbf{F}}(t) = \sin(t)\mathbf{i} + 3t^2\mathbf{j} + e^t\mathbf{k}$$

For various values of t, we'll get different vectors, i.e., $\vec{\mathbf{F}}(0) = 3\mathbf{j} + \mathbf{k}$ and $\vec{\mathbf{F}}(\frac{\pi}{2}) = \mathbf{i} + \frac{3\pi^2}{4}\mathbf{j} + e^{\frac{\pi}{2}}\mathbf{k}$.

The functions $x(t), y(t), \text{and } z(t)$ are the *component functions*, or *components*, of $\vec{\mathbf{F}}(t)$. We call $\vec{\mathbf{F}}$ *continuous* if each component function is continuous. For example, $\vec{\mathbf{F}}(t) = \sin(t)\mathbf{i} + 3t^2\mathbf{j} + e^t\mathbf{k}$ is continuous for all *t* while $\vec{\mathbf{F}}(t) = 3\mathbf{i} + \frac{2}{t}\mathbf{j} + \ln(t)\mathbf{k}$ is continuous for t > 0.

We say that $\vec{\mathbf{F}}(t)$ is *differentiable* if each component function is differentiable. If so, we define the first derivative as

$$\vec{\mathbf{F}}'(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j} + z'(t)\mathbf{k}$$

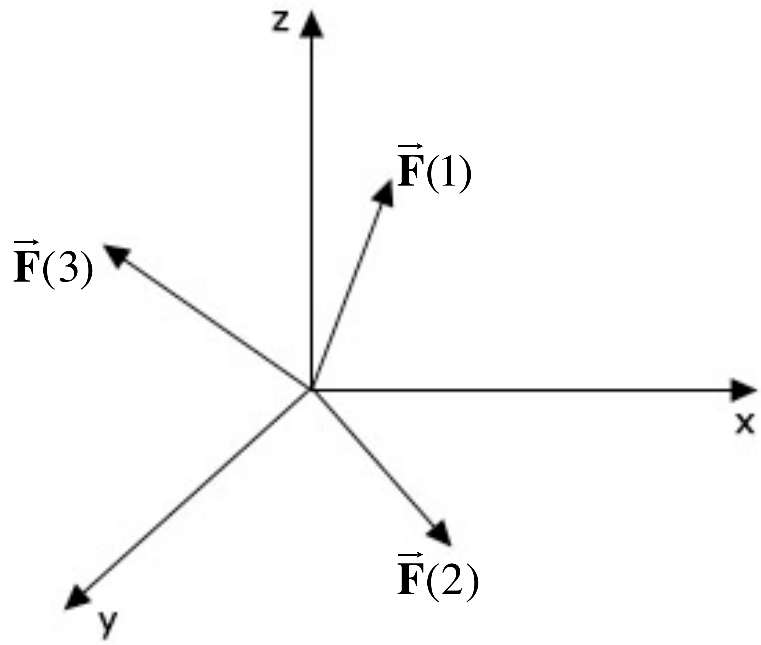
Note that by differentiating a vector function, we get another vector function. As with most real value functions, a vector function may not be differentiable. Consider

$$\vec{\mathbf{G}}(t) = |t|\mathbf{i} + \frac{1}{t}\mathbf{j}$$

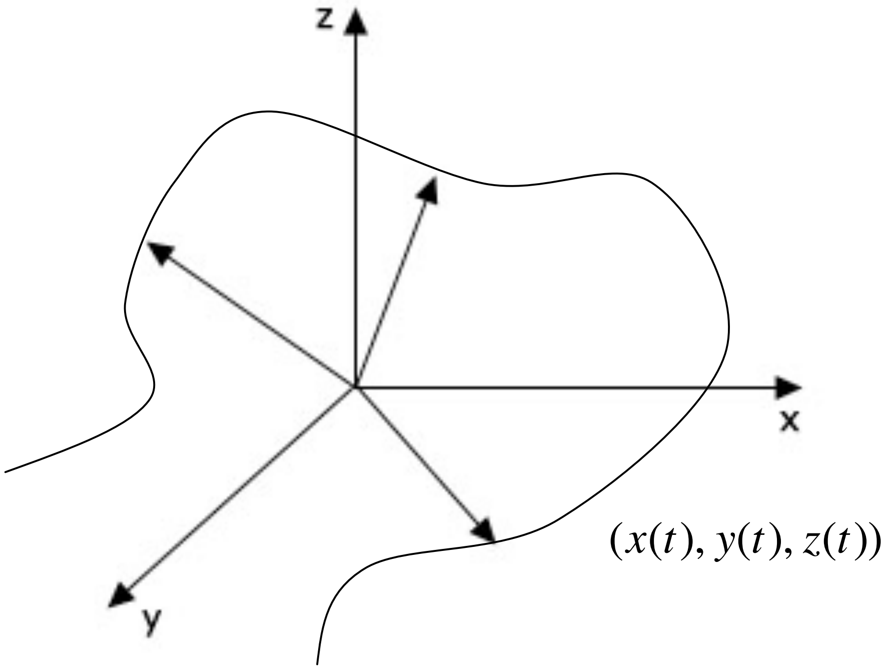
While $\vec{\mathbf{G}}(t)$ is continuous for all *t*, we can't differentiate it because one of its component functions $|t|$ is not differentiable and so $\vec{\mathbf{G}}(t)$ is not differentiable.

Now that you got a rough feel of vector functions, here comes the most important part. You need to divorce, well at least the geometrical aspect, the vector functions from the real-value functions in term of its representation on a graph. See, for a real-value function, you pick a value for x, get a corresponding for y and plot the point on the graph. For vectors, the vector function gives you a VECTOR, an arrow in the space which brings you from a point to another point. The starting point is the origin and the ending point is the point where the vector brings you. Using different values of the parameter gives you different vectors.

And so, geometrically, we may envision a vector function as an adjustable arrow pivoted at the origin.




$\vec{\mathbf{F}}(t)$ represents the arrow from the origin to the point $(x(t), y(t), z(t))$ and so for different values of *t*, we get different vectors of $\vec{\mathbf{F}}(t)$. As *t* varies, the arrow sweeps out a curve in space. This curve is the locus of points $(x(t), y(t), z(t))$.




At first sight, it seems that the vector function does give us a curve. Yes ultimately it does, and the term curve will also be used in vector calculus. But it is vitally important to know that this curve is traced out by vectors from the vector function. Sometimes this curve is called a *trajectory* and the vector $\vec{\mathbf{F}}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ is called the *position vector* of this curve.

So remember, a curve in vectors are the points which vectors from vector function travel to from the origin.