Fluid Mechanics

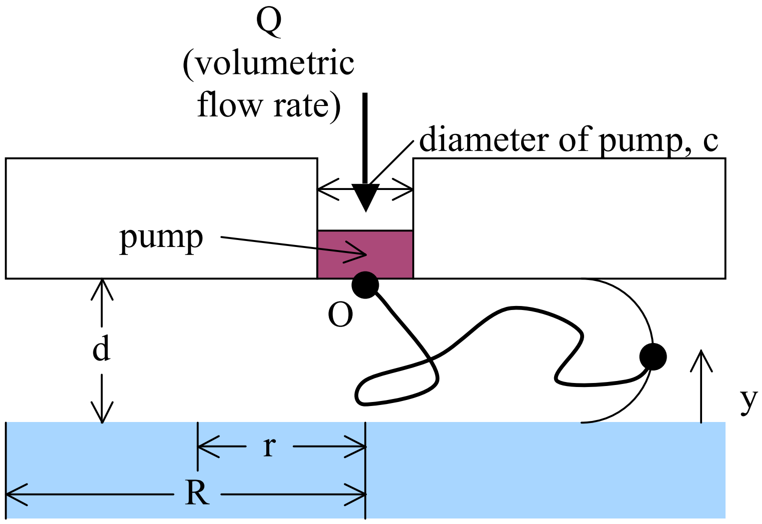
Hovercraft Revisited

With our newly acquired knowledge of Bernoulli's equation, we are prepared to extend our analysis of the Hovercraft. This lesson, appropriately named 'Hovercraft Revisited' is about finding the force it takes to suspend the hovercraft above the water.

The crux of the problem is get the pressure of the air as it exits the pump in the radial direction. We suspect that this pressure is dependent on the radius from the centre and the depth from the water surface, which it actually turns out as such.

Application of Bernoulli's equation begins by drawing a streamline connecting the two points we wish to analyze. One point of our analysis a certain volume of the air along the radial direction in terms of *r*. However, where does our streamline start?

It would be a mistake to draw a streamline from a point before the pump. Remember Bernoulli's equation is a representation of the conservation of mechanical energy and the streamline going through the pump disobeys this as the pump adds mechanical energy. Instead, we will look at the streamline from the point after the pump and to the volume of air at a distance *r*, as shown below.



The Bernoulli equation is given as

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}\,[\,= P_*\,]$$

Let O be the location shortly after the pump. The velocity at O is related to the volumetric flow rate by

$$Q = \pi r^2 U_0$$
$$U_0 = \frac{Q}{\left(\pi \frac{c^2}{4}\right)}$$

since the diameter is given by *c*. Hence applying Bernoulli's equation to point O gives us

$$P_0 + \frac{1}{2}\rho\left[\frac{Q}{\left(\pi c^2/4\right)}\right]^2 + \rho g y = P_*$$

Now let's look at any point in the radial location *r* along the gap, in particular the terms for pressure and velocity. As seen from the diagram drawn previously, you could be quick to conclude the pressure as P_{atm} as the layer of air is in contact with the free surface. This is NOT the case. We are looking at a cross-section area of whole layer of air in the gap. The air that flows out forms a parabolic velocity profile which is constantly flowing. Moreover, if the pressure is P_{atm} for whatever value of *r*, the velocity of the air is the same. This is not the case as shown from our previous lesson. Instead, we label this pressue as $P(r)$ as it changes as *r* changes.

How about the velocity term. By recalling that volume of air enters the control volume with a volumetric flow rate of *Q*. However, this volume of air is exiting the control volume through the control surface at the same rate. Therefore, our velocity is given by

$$U = \frac{\text{Rate of volume of air flowing out}}{\text{Area of control surface}}$$
$$U = \frac{Q}{2\pi r d}$$

Again, noticing that the velocity is dependent on *r* and *d*. Now we can apply the equation.

At any radial location *r* along the gap,

$$P(r)+\frac{1}{2}\rho U^2+\rho g y=P_*$$

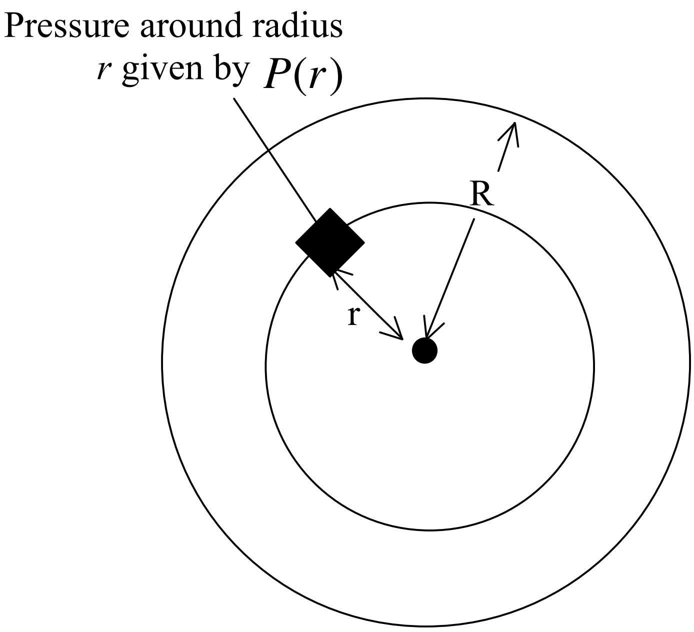
$$P(r)+\frac{1}{2}\rho\left(\frac{Q}{2\pi r d}\right)^2+\rho g y=P_*$$

Equating the two and rearranging, we get,

$$P(r)+\frac{1}{2}\rho\left(\frac{Q}{2\pi r d}\right)^2+\rho g y=P_0+\frac{1}{2}\rho\left[\frac{Q}{\left(\pi c^2/4\right)}\right]^2+\rho g y$$

$$P(r)=P_0+\rho\frac{Q^2}{\pi^2}\left(\frac{8}{c^4}-\frac{1}{8r^2d^2}\right)$$

With this pressure function, we can calculuate the required force to suspend the hovercraft. The pressure function is dependent on *r*, thus we need to consider a ring of radius *r* where the pressure is given by $P(r)$.



Top view of hovercraft

Recalling that force is equal to pressure multiplied by area, in this case is $2\pi r dr$ for a small area of constant *r*, the required force is given by

$$F = \int_0^R P(r) 2\pi r dr = 2\pi \int_0^R P_0 + \rho \frac{Q^2}{\pi^2} \left(\frac{8}{c^4} - \frac{1}{8r^2 d^2} \right) r dr$$

which we would have to refer to a table of integrals to find the final result.

And there we go, using Bernoulli's equation to derive the pressure function and then using it to calculate the required force to suspend the hovercraft. This problem is a personal favourite of mine.