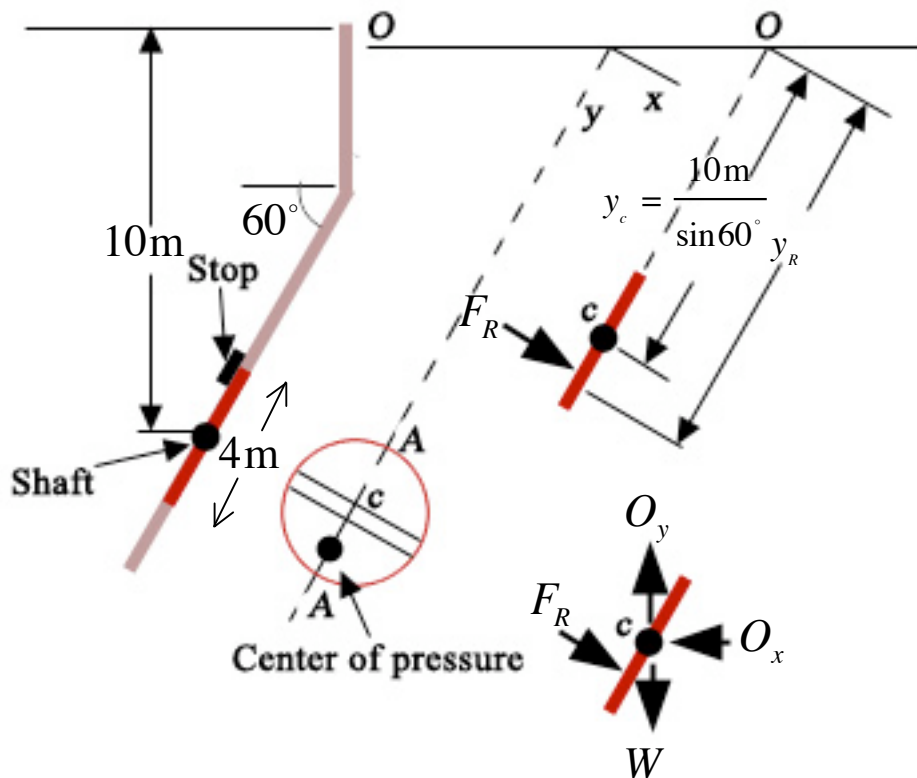


Fluid Mechanics

**Example – Hydrostatic Force on a gate**

A 4m diameter circular gate, mounted on a shaft along its horizontal diameter, is located on an inclined wall of a large reservoir containing water  $\gamma = 9.8 \text{ kN} / \text{m}^3$ . For a depth of 10m above the shaft determine, 1. the magnitude and location of the resultant force exerted on the gate by the water, and 2. the moment that would have to be applied to open the gate.

This is a fairly straightforward problem of using the formulas to find the magnitude and location of the hydrostatic force and then taking moments about a point. Our problem is illustrated below:



Magnitude of force is given by

$$F_R = \gamma h_c A$$

using vertical distance from the fluid surface to the centroid of the area given by 10m, it follows that

$$\begin{aligned} F_R &= (9.80 \times 10^3 \text{ N} / \text{m}^3)(10 \text{ m})(4\pi \text{ m}^3) \\ &= 1230 \times 10^3 \text{ N} = 1.23 \text{ MN} \end{aligned}$$

To find the location of the force, or center of pressure, through which  $F_R$  acts, we use equations

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \quad y_R = \frac{I_{xc}}{y_c A} + y_c$$

From our coordinate system,  $x_R = 0$  since the area is symmetrical, and the centre of pressure must lie along the diameter A-A. To find  $y_R$ , we use the standard results of moment of inertia,

$$I_{xc} = \frac{\pi R^4}{4}$$

$$\begin{aligned} y_R &= \frac{(\pi/4)(2 \text{ m})^4}{(10 \text{ m} / \sin 60^\circ)(4\pi \text{ m}^2)} + \frac{10 \text{ m}}{\sin 60^\circ} \\ &= 0.0866 \text{ m} + 11.55 \text{ m} = 11.6 \text{ m} \end{aligned}$$

and the distance along the gate below the shaft to the centre of pressure is

$$y_R - y_C = 0.0866 \text{ m}$$

We shift our attention to the moment created by the respective forces. The moment required to open the gate is facilitated with the help of a free-body diagram.  $W$  is the weight of the gate and  $O_x$  and  $O_y$  are the horizontal and vertical reactions of the shaft on the gate. Summing moments about the shaft,

$$\sum M_c = 0$$

and therefore

$$\begin{aligned} M &= F_R (y_R - y_C) \\ &= (1230 \times 10^3 \text{ m})(0.0866 \text{ m}) \\ &= 1.07 \times 10^5 \text{ Nm} \end{aligned}$$

The moment required to open the gate is  $1.07 \times 10^5$  Nm, the moment needed to oppose the one from the hydrostatic force.