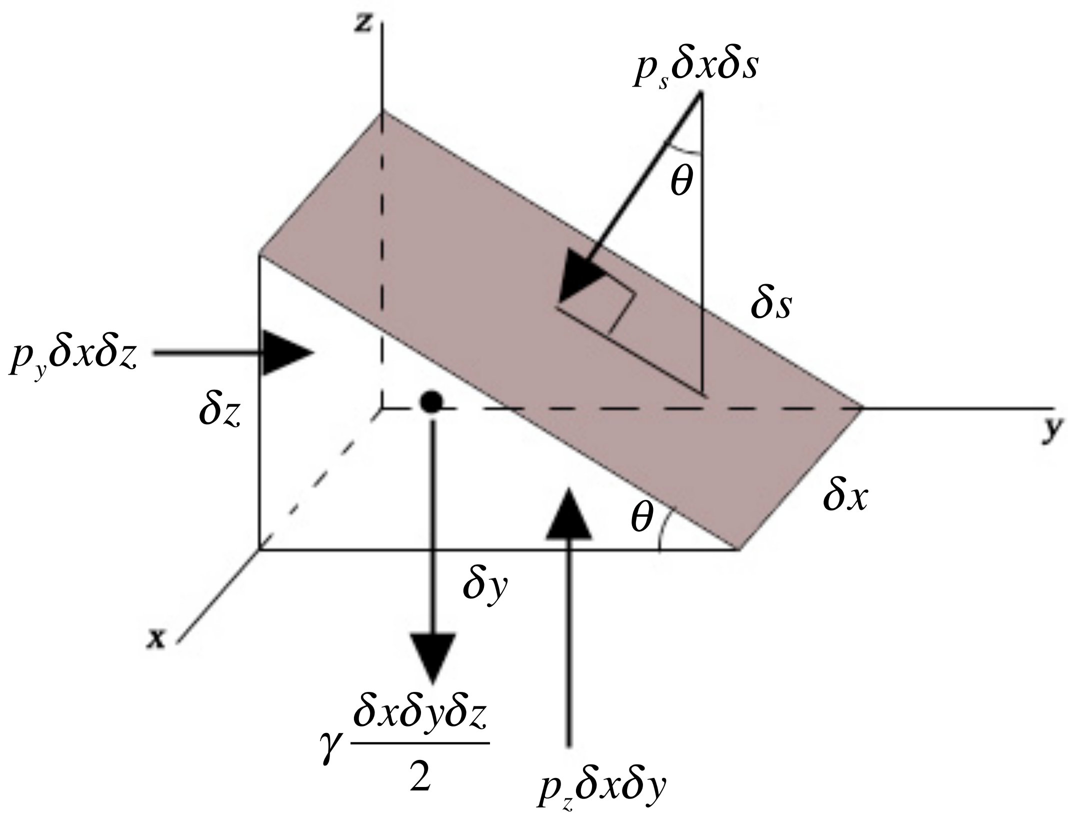
Fluid Mechanics

Pressure at a point

Pressure is used to indicate the normal force per unit area at a given point acting on a given plane within the fluid mass of interest. The common question which arises is how this pressure varies as we change the orientation of the plane for which the passing through this point. We will answer that question here.

In the fluid, we pick out a small triangular wedge of the fluid from some arbitrary location within the fluid mass. With the assumption of no shearing stress between particles in the liquid, we analyze the external forces acting on the wedge due to pressure on its planes and its own weight. For simplicity, the forces in the *x* direction are not shown. The vertical axis is labeled as *z* so the weight acts in the negative *z* direction. As you can see, the arbitrary value of θ means that the slant of this wedge shape is allowed to vary to take the form of any triangular shape. We'll allow the fluid element to have accelerated motion though the assumption of zero shearing stresses will still be valid so long as the fluid element moves as rigid body.



Applying Newton's second law, $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, in the *y* and *z* directions are respectively

$$\sum F_y = p_y \delta x \delta z - p_s \delta x \delta s \sin\theta = \rho \frac{\delta x \delta y \delta z}{2} a_y$$

$$\sum F_z = p_z \delta x \delta y - p_s \delta x \delta s \cos\theta - \gamma \frac{\delta x \delta y \delta z}{2} = \rho \frac{\delta x \delta y \delta z}{2} a_z$$

where p_s, p_y, p_z are the average pressures on the faces, γ denotes specific weight, that is the force due to gravity per mass, and ρ denotes density. Please be reminded that a pressure must be multiplied by an appropriate area to obtain the force generated by that pressure. From geometry, we see that

$$\delta y = \delta s \cos\theta$$
$$\delta z = \delta s \sin\theta$$

allowing us to write the equations of motion as

$$p_y - p_s = \rho a_y \frac{\delta y}{2}$$

$$p_z - p_s = (\rho a_z + \gamma) \frac{\delta z}{2}$$

Since we are interested at what is happening at a point, we take the limit as $\delta x, \delta y, \delta z$ approach zero, it follows that

$$p_y = p_s, \quad p_z = p_s$$

or more commonly written as

$$p_s = p_y = p_z$$

The angle θ is still arbitrary chosen so we can conclude that *the pressure at a point in a fluid at rest, or in motion, is independent of the direction as long as there is no shearing stresse present*. This result is otherwise known as *Pascal's law* named in honor of Blaise Pascal (1623–1662), a French mathematician who made important contributions in the field of hydrostatics. This would have been a standard result in physics. We have proven it mathematically in here.