Fluid Mechanics

Pressure Variation for Fluid at Rest – Compressible

For gases such as air, oxygen, and nitrogen, they are usually compressible meaning that the density of the gas changes significantly with pressure and temperature. This needs to be taken into consideration when finding the pressure equation. We shall look at it here.

Following up from our previous equation, it is necessary to consider the possible variation in γ before the equation can be integrated. While we already know what compressible gases are, it is equally important to note that since specific weights of gases are comparatively small compared to liquids, it follows from

$$\frac{dp}{dz} = -\gamma$$

that the pressure gradient in the vertical direction is correspondingly small, or even over distances of several hundred feet the pressure will remain essentially constant for a gas. This implies that we can neglect the effect of elevation changes on the pressure in gases in tanks, pipes, in which the distances involved are small.

Should the variations in heights be large, on the order of thousands of feet, attention must be given to the specific weight. Our way around this is to use the equation of state for an ideal gas.

$$p = \rho RT$$

where *p* is the absolute pressure, *R* is the gas constant, and *T* is the absolute temperature. Combing these equations, we get

$$\frac{dp}{dz} = -\frac{gp}{RT}$$

and by separating variables

$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln\frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T}$$

where *g* and *R* are assumed to be constant over the elevation change from z_1 and z_2. Changes in *g* are usually small and so we assumed it to be constant at some average value for the range of elevation involved.

We still have one more condition to set. Notice that temperature *T* may also vary due to the change in height. Hence, we assume the temperature to be a constant T_0 over the range z_1 and z_2. This is called an *isothermal* condition, quite often used in analysis of liquids. By integrating, we get

$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

This equation provides the desired pressure-elevation relationship for an isothermal layer.