

Fluid Mechanics

Pressure Variation in Fluid with Rigid-Body Motion

We'll now focus our analysis on fluids which are in motion. The equation used is the basic equation of pressure variation, but this time, we will factor in the linear motion of the liquid.

The basic equation is

$$-\nabla p - \gamma \hat{\mathbf{k}} = \rho \vec{\mathbf{a}}$$

No shearing stress between the liquid particles is still assumed. We can write this equation based on rectangular coordinates with the positive z axis being vertically upwards as

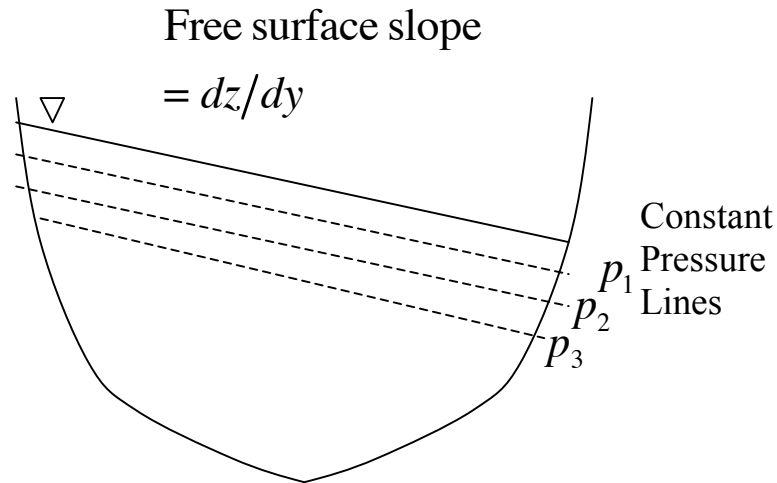
$$-\frac{\partial p}{\partial x} = \rho a_x$$

$$-\frac{\partial p}{\partial y} = \rho a_y$$

$$-\frac{\partial p}{\partial z} = \gamma + \rho a_z$$

In fluid mechanics, there is a general class of problems involving fluid motion in which there are no shearing stresses when a mass of fluid undergoes rigid-body motion. This means that the fluid will move as a rigid mass with each particle having the same acceleration. A common example is a cup of water in an accelerating vehicle. No deformation implies no shearing stress and so the basic equation is valid. From here, we will develop the equation of the pressure field for a fluid under linear motion.

Consider an open container of a liquid that is translating along a straight path with a constant acceleration $\vec{\mathbf{a}}$ as illustrated below.

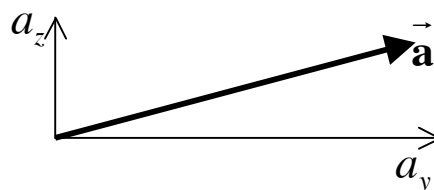


For simplicity, we taking that $a_x = 0$ and from the previous differential equations, $\partial p / \partial x = 0$. This leaves us with

$$\frac{\partial p}{\partial y} = -\rho a_y$$

$$\frac{\partial p}{\partial z} = -\rho(g + a_z)$$

which relates the acceleration component in the y and z axis with the respective change in pressure along that axis. Note we have broken down the acceleration vector into these two components.



The change in pressure between two closely spaced points located at y, z , and $y + dy, z + dz$ can be expressed as

$$dp = \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz$$

and by substituting,

$$dp = -\rho a_y dy - \rho(g + a_z) dz$$

Along a line of *constant* pressure, $dp = 0$ and therefore it follows that the slope of this line is given by the relationship

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z}$$

Along a free surface the pressure is constant, so that for the accelerating mass shown previously, the free surface will be inclined if $a_y \neq 0$. Moreover, all lines of constant pressure will be parallel to the free surface as illustrated.

For the circumstance in which $a_y = 0$, $a_z \neq 0$, which corresponds to the mass of fluid accelerating in the vertical direction, this equations indicates that the fluid surface will be horizontal. However, from

$$\frac{\partial p}{\partial z} = -\rho(g + a_z)$$

we see that the pressure distribution is not hydrostatic, but is given by the equations:

$$\frac{dp}{dz} = -\rho(g + a_z)$$

This tells us that for fluids of constant density, the pressure will vary linearly with depth, but the variation is due to the combined effects of gravity and the externally induced acceleration, $\rho(g + a_z)$ rather than simply the specific weight ρg . This gives rise to two common situations.

One, the pressure along the bottom of a liquid-filled tank which is resting on the floor of an elevator that is accelerating upward will be increased over that which exists when the tank is at rest.

Two, for a *freely falling* fluid mass ($a_z = -g$), the pressure gradients in all three coordinate directions are zero, which means that if the pressure surrounding the mass is zero, the pressure throughout will be zero.