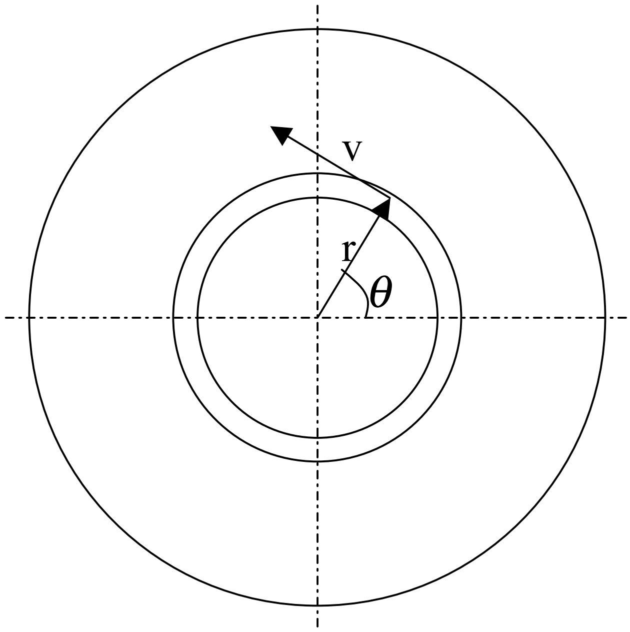
Fluid Mechanics

Shearing Stress on a Disk

Using the concept of viscosity and shearing stress, let us look at a typical problem which applies that concept.

We have a disk, diameter 200mm, rotating against a bottom stationary disk. The two surfaces are separated by a very thin uniform oil film of δ thick and the top disk is rotating at Ω rpm.

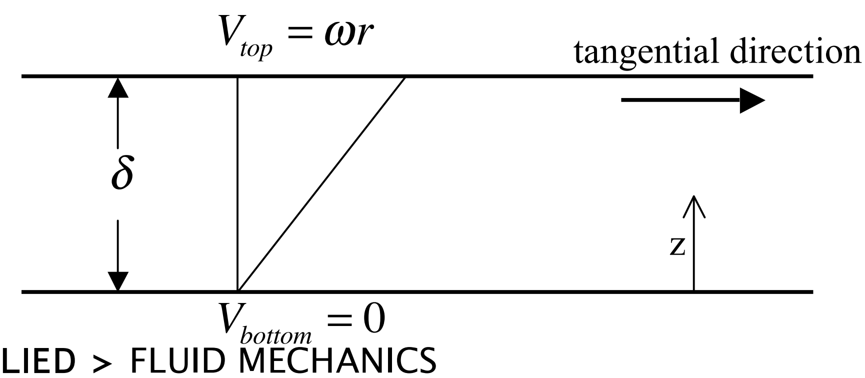
We want to first wish to find the gradient of the tangential velocity at the bottom surface. When view from the top, the disk looks like this.



Using equations of circular motion, we find that the tangential velocity is given by

$$v_T = \omega r \text{ where } \omega = \frac{2\pi\Omega}{60} \text{rads}^{-1}$$

It is important that we realize this is the tangential velocity of the top plate. Hence, if you were to view the disk from the side,



Further, notice that the tangential velocity *is* a function of *r*. This makes the calculations a little more involved but still easily manageable. If we assume a linear tangential velocity profiles and that the velocity at the bottom is 0, the tangential velocity gradient is given by

$$\frac{dv}{dz} = \frac{\omega r}{\delta}$$

We then use this quantity to find the shearing stress at radius *r*. Notice we still have to keep functions in terms of *r* as the shearing stress is simply different at different radiuses.

$$\tau_{z\theta} = \mu \frac{dv}{dz} = \mu \frac{\omega r}{\delta}$$

We denote $\tau_{z\theta}$ as the shearing stress in the tangential θ direction.

Next up is to calculate the tangential shear force. By definition, the shearing is given by "Shear Force = Shear stress x area where the shear stress acts." Here is where some calculus comes in. See, the shear stress is dependent on the radius; hence we need to consider the shear force at radius *r* to radius (*r* + *dr*), the area concerned. That is

$$dF = \tau_{z\theta}(r) \cdot 2\pi r\, dr$$

Total shear force is now

$$F = \int dF = \int_0^R \tau_{z\theta} \cdot 2\pi r\, dr = \frac{2\mu\omega\pi R^3}{3\delta}$$

Our last calculation is the power absorbed by the lower surface. See, the top rotating disk is doing work on the fluid and this gets absorbed by the lower surface. Start by considering a small work done on a small strip,

$$dW = dF \times v_T$$

$$W = \int_0^R \frac{\mu\omega r}{\delta} 2\pi r \omega r\, dr$$

$$= \frac{\pi\mu\omega^2 R^4}{2\delta}$$

I must say that this isn't a standard shearing stress problem. What mainly makes this question different is that the velocity and hence the shearing stress is depends on another variable, that is *r*, the radius from the centre. Once you get this idea, the calculations should smoothly follow.