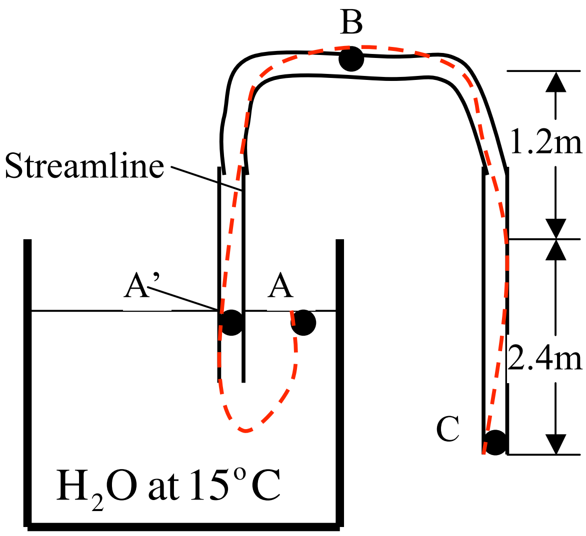
Fluid Mechanics
Siphoning the Water

Any student who has every done a physics practical will be familar with the phenomena of siphoning water, you know the unusual process of clearing a beaker of water by channeling the water up a tube which seems to defy gravity. We shall explain how it is done.

A siphon is shown in the figuare below. Assuming that fluid viscosity can be neglected, we seek to estimate the velocity of water leaving C as a free jet. In addition, we later find the pressure of water in the tube at B and at A'. We assume that the flow is steady and the water is incompressible.



We draw a streamline as labeled by the red dotted line from the free surface at point A to the free jet at point C. Applying Bernoulli's equation between these two points, we have

$$\frac{P_{atm}}{\rho}+\frac{V_C^{\,2}}{2}+g(z_C=0)=\frac{P_{atm}}{\rho}+\frac{V_A^{\,2}}{2}+g(z_A=2.4)$$

First we noticed that at both points, the water is exposed to the atmosphere and thus the pressure at each point is equal. We have taken the reference level to be at point C as shown $z_C=0$. To proceed further, we make some logical estimates.

Since the flow rate of water is through a small tube from a large container, we can say that $V_A \ll V_C$ or equivalently $V_A \approx 0$. Just picture the water level of the container decreasing very slowly.

Our equation now becomes

$$\frac{V_C^{\ 2}}{2} = g \cdot 2.4$$

and so $V_C = 6.86\text{ms}^{-1}$.

We do the same of drawing a streamline and applying Bernoulli's equation, this time from A' inside the tube to point C. Again, we take C as the reference level.

$$\frac{P_{A'}}{\rho} + \frac{V_{A'}^{\ 2}}{2} + 2.4g = \frac{P_{atm}}{\rho} + \frac{V_C^{\ 2}}{2}$$

Beware that the pressure at point A' is NOT atmospheric pressure simply because the water is moving with a certain velocity in the tube, unlike the water at point A. From the principle of continuity, $V_{A'} = V_C$ and so

$$P_{A'} - P_{atm} = -2.4 \times 9.81 \times \rho = -23.5\,\text{kPa}$$

This is the difference between the pressure at A' and the atmosphere. We can simply use a given value for atmospheric pressure to find $P_{A'}$.

Lastly, doing the same for points B and C

$$\frac{P_B}{\rho} + \frac{V_B^{\ 2}}{2} + 2.6g = \frac{P_{atm}}{\rho} + \frac{V_C^{\ 2}}{2}$$

Again, from the principle of continuity, $V_B = V_C$, and so

$$P_B - P_{atm} = -3.6 \times 9.81 \times \rho = -35.3\,\text{kPa}$$

The term $P - P_{atm}$ is called the gauge pressure at a certain point. While the values of gauge pressure at A' and B are negative, we are more concern with the relative difference between the two. Notice that the gauge that A' is more (less negative) than that at B. This is the reason why water is able to travel up the tube.