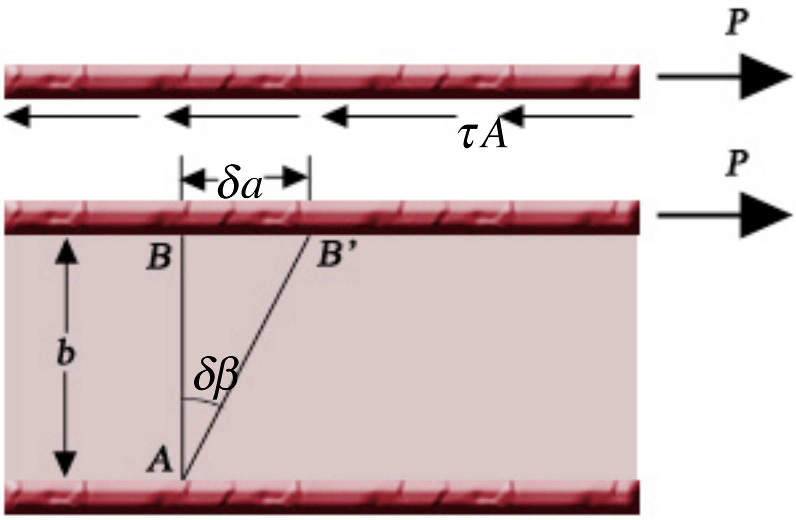
Fluid Mechanics
Viscosity and Shearing Stress

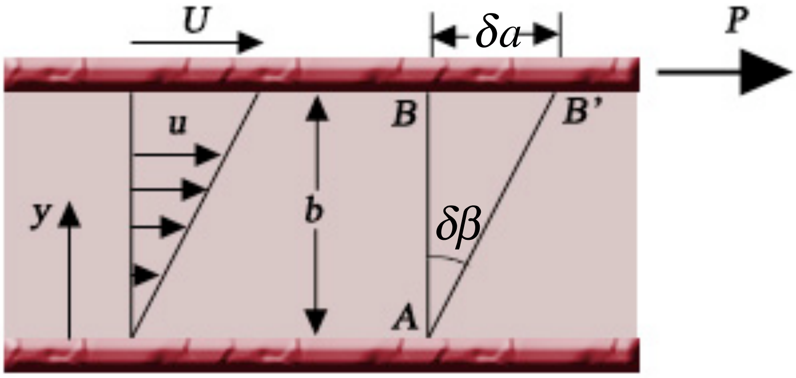
Properties such as specific weight and density measure the "heaviness" of a fluid. It is highly possible that two liquids with the same specific weight and density have a different flow pattern. This motivates us to find another quantity to describe the "fluidity" of the fluid.

Let's think of a hypothetical experiment that sheds some meaning to this property. We place a liquid between two very wide parallel plates. The bottom plate is rigidly fixed but the upper plate is free to move. When the top plate is moved with a force *P*, the top plate will move continuously with a velocity *U* as illustrated below.



There has to be a resistance to the movement caused by the "fluidity" of the liquid, roughly speaking. We say that a shearing stress, τ is developed at the plate-liquid interface. Our objective is thus to find a mathematical expression for τ.

This shearing stress acts on the plate and by Newton's third law, it also acts on the liquid, causing it to deform continuously. Upon closer inspection, the upper plate moves with the pate velocity, *U*, and the fluid in contact with the bottom plate has zero velocity. The fluid between the two plates would move with the velocity govern by the velocity function $u = u(y)$ or if found to vary linearly, $u = Uy/b$. We say that a *velocity gradient*, du/dy is developed in the fluid between the pates. This gradient may or may not be constant depending on the complexity of the fluid.



Time to introduce some variables. In a small time increment, δt, an imaginary vertical line *AB* in the fluid would rotate through an angle, $\delta\beta$, so that

$$\tan\delta\beta \approx \delta\beta = \frac{\delta a}{b}$$

Since $\delta a = U\delta t$, it follows that

$$\delta\beta = \frac{U\delta t}{b}$$

At this point $\delta\beta$ is a function not only of the force *P* but also of time. It doesn't seem reasonable to relate the shearing stress, τ to $\delta\beta$. A way around this is to consider the *rate* at which $\delta\beta$ is changing, and define the *rate of shearing strain*, $\dot{\gamma}$, as

$$\dot{\gamma} = \lim_{\delta t \to 0} \frac{\delta\beta}{\delta t}$$

$$\dot{\gamma} = \frac{U}{b} = \frac{du}{dy}$$

From experimental results, we conclude that the shearing stress, τ, increases in direct proportion to the shearing strain, that is

$$\tau \propto \dot{\gamma}$$

$$\tau \propto \frac{du}{dy}$$

This result indicates that for common fluids such as water, oil and air the shearing stress and rate of shearing strain (velocity gradient) can be related with the relationship of the form

$$\tau = \mu \frac{du}{dy}$$

where the constant of proportionality is designated by the Greek symbol μ (mu) called the *absolute viscosity*, *dynamic viscosity*, or simply the *viscosity* of the fluid. The actual value of the viscosity depends on the particular fluid, and it should be noted that for a particular fluid, the viscosity is highly depended on the temperature.

Depending on convention, I usually talk about the shearing stress *on* the fluid. While there is no strict rule to this, I leave it to the reader to understand the concept of shearing stress and that by Newton’s third law, the shearing stress acting on the fluid and on the plate are equal and opposite and on different bodies.