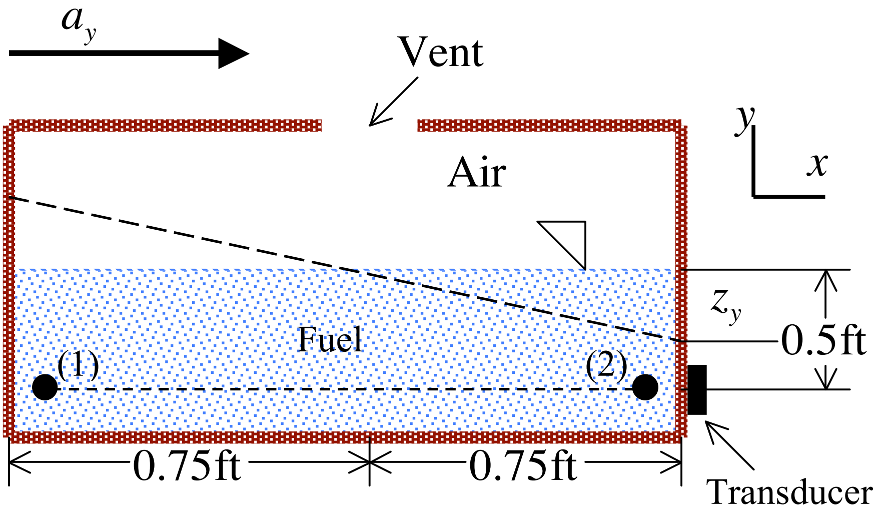
Fluid Mechanics

Example – Water in a cup

You have probably brought a cup of water, or any drink for that matter, in a car and see the level of the water change as the car moves, or accelerates to be more exact. This example is an explanation of that phenomenon, using a more serious example of a fuel tank but the principle is still the same.

We have a rectangular tank that is vented at the atmosphere and a pressure transducer is located in its side as illustrated. The tank is placed in an experimental vehicle and is subjected to a constant linear acceleration, a_y.



We want to determine an expression that relates a_y and the pressure at the transducer for a fuel with a *SG=0.65*. In addition, for this experiment, the transducer serves as a warning when the water level drops below it. We want to find the maximum acceleration when this occurs.

For constant horizontal acceleration, the fuel will move as a rigid body. Using our pressure-gradient equation from the previous lesson bearing in mind that $a_z = 0$,

$$\frac{dz}{dy} = -\frac{a_y}{g}$$

For some arbitrary a_y, the change in depth, z_1 of liquid on the right side of the tank can be found by substituting the values from the diagram.

$$-\frac{z_1}{0.75\text{ft}} = -\frac{a_y}{g}$$

$$z_1 = 0.75\text{ft}\cdot\left(\frac{a_y}{g}\right)$$

Since there is no acceleration in the vertical, *z* direction, the pressure along the wall varies hydrostatically. Thus, the pressure at the transducer is given by the relationship

$$p = \gamma h$$

where *h* is the depth of fuel above the transducer, and so

$$\begin{aligned} p &= SG\cdot\rho_{water}\cdot[0.5\text{ft} - (0.75\text{ft})(a_y/g)] \\ &= 0.65\cdot 62.4\text{lbft}^{-3}\cdot[0.5\text{ft} - (0.75\text{ft})(a_y/g)] \\ &= 20.3 - 30.4\cdot\frac{a_y}{g} \end{aligned}$$

for $z_1 \le 0.5\text{ft}$. A few things to note there. We are given the specific gravity of the fuel, hence we need to multiply it by the density of water to get the density of the fuel. Second, notice that height is measure by the height of fuel above the transducer and not by the depth from the free surface. This makes sense as the *h* used in the equation is the depth of the *fuel*. If you measure it from the free surface, there is no fuel.

The limiting value for a_y (when the fuel level reaches the transducer) can be found from the equation

$$0.5\text{ft} = 0.75\text{ft}\cdot\left(\frac{(a_y)\text{max}}{g}\right)$$

$$(a_y)\text{max} = \frac{2g}{3}$$

and using standard acceleration for gravity,

$$(a_y)\max = \frac{2}{3} \cdot 32.2\text{fts}^{-2} = 21.5\text{fts}^{-2}$$

We also enforce here that the pressure in horizontal layers is not constant in this example since

$$\frac{\partial p}{\partial y} = -pa_y \neq 0$$

implying that the line from (1) to (2) is *not* a line of constant pressure. Instead, the slanted line is of constant pressure.

In fluid mechanics problems, you will face problems which are similar to this. Thus, it is good to absorb the technique to solve such a problem. Moreover, now to can tell your friends in the car the slope of the water, not that it matters to them though.