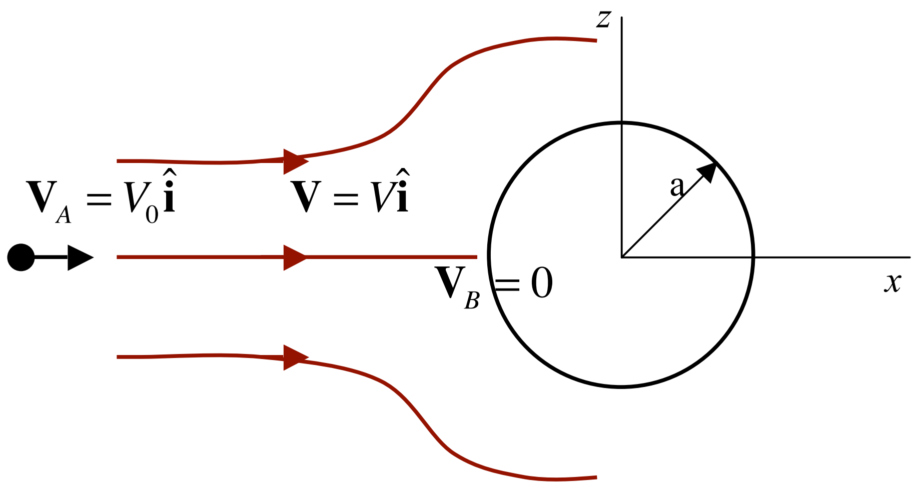
Fluid Mechanics

Water flowing to a sphere

We shall now use Bernoulli's equation to solve a problem of water flowing to a sphere. The objective here to analyze the change in pressure of the water stream.

Consider the inviscid, incompressible, steady flow along the horizontal streamline A–B in front of the sphere of radius *a* as shown below.



Advance theory of fluid flow past a sphere tells use that the fluid velocity along this streamline is given by

$$V = V_0\left(1 + \frac{a^3}{x^3}\right)$$

We need not be concerned with the derivation of this result, which could have been calculated from experimental data. Instead let us determine the pressure variation along the streamline from point A infinitely far in front of the sphere where $x_A = -\infty$ and $V_A = V_0$, to point B where $x_B = -a$ and $V_B = 0$.

We start our analysis from the Bernoulli's equation of the form:

$$-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$$

While we could have used the more general form of $p + \frac{1}{2}\rho V^2 + \gamma z = C$, notice that we need differential quantities to find the change in pressure. Technically, we could have reached the required form by differentiating the general form. Since the streamline is horizontal, $\sin\theta = \sin 0 = 0$ and the equation of motion reduces to

$$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s}$$

With the given velocity variation along the streamline, we rewrite the acceleration term as

$$V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial x} = V_0\left(1+\frac{a^3}{x^3}\right)\left(-\frac{3V_0 a^3}{x^4}\right)$$
$$= -3V_0^{\ 2}\left(1+\frac{a^3}{x^3}\right)\frac{a^3}{x^4}$$

where we have replaced *s* by *x* since the two coordinates are identical along streamline A–B. It follows that $V\,\partial V/\partial s < 0$ along the streamline. The fluid slows down from V_0 far ahead of the sphere to zero velocity on the 'nose' of the sphere at $x = -a$.

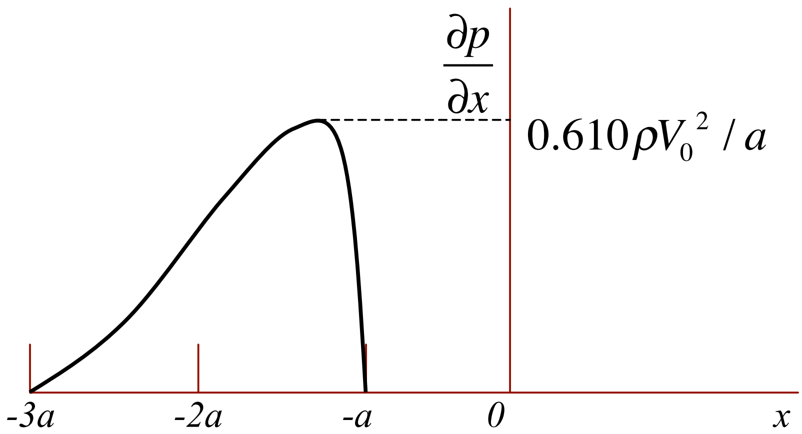
Substituting into our Bernoulli's equation, the pressure gradient to produce the given motion along the streamline is

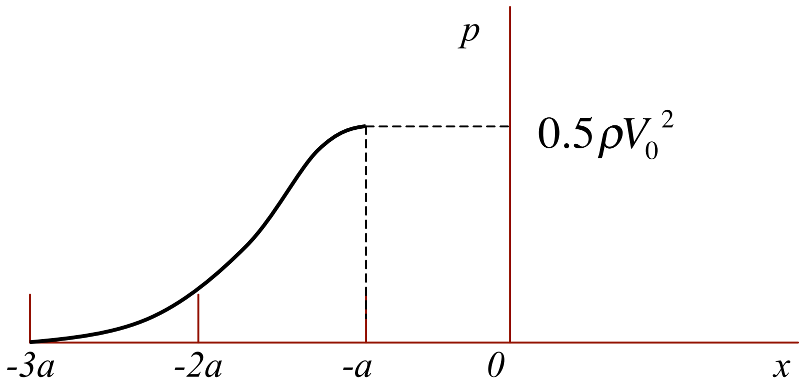
$$\frac{\partial p}{\partial x} = \frac{3\rho a^3 V_0^{\ 2}(1 + a^3/x^3)}{x^4}$$

The variation pressure is indicated on the graph as shown. It is seen that the pressure increases in the direction of flow $(\partial p/\partial x > 0)$ from point *A* to point *B*. The maximum pressure gradient is $0.610\rho V_0^{\ 2}/a$ which occurs just slightly ahead of the sphere at $x = -1.205a$. It is the pressure gradient that slows the fluid down from V_0 to 0.

We can get the pressure distribution along the streamline by integrating the previous equation from $p = 0$ at $x = -\infty$ to pressure p at location x giving us

$$p = -\rho V_0^{\ 2}\left[\left(\frac{a}{x}\right)^3 + \frac{(a/x)^6}{2}\right]$$





The pressure at *B* and $V_B = 0$ is the highest pressure along the streamline $p_B = \rho V_0^{\ 2} / 2$, by inspecting that we get a maximum value by substituting $x = -a$ into the pressure equation. This excess pressure on the front of the sphere contributes to the net drag force on the sphere.

Lastly, the pressure gradient and pressure are directly proportional to the density of the fluid, implying that the fluid inertia is proportional to its mass.