Fourier Analysis

Example of a Fourier Series on [−*L*, *L*]

With our new definition of a Fourier Series on [−*L*, *L*], let us looking at a simple example. In the later lessons, functions can be either defined from $[-\pi, \pi]$ or [−*L*, *L*]. Please use the appropriate definitions.

Let

$$f(x) = \begin{cases} 0 & \text{for} -3 \le x \le 0 \\ x & \text{for } \ 0 \le x \le 3 \end{cases}$$

Our task is to find the Fourier series of *f* on [−3, 3].
With *L* = 3, compute

$$a_0 = \frac{1}{6}\int_{-3}^{3} f(x)dx = \frac{1}{6}\int_{0}^{3} xdx = \frac{3}{4},$$

$$a_n = \frac{1}{3}\int_{-3}^{3} f(x)\cos\left(\frac{n\pi x}{3}\right)dx$$

$$= \frac{1}{3}\int_{0}^{3} x\cos\left(\frac{n\pi x}{3}\right)dx = \frac{3}{n^2\pi^2}\left[\cos(n\pi) - 1\right]$$

and

$$b_n = \frac{1}{3}\int_{-3}^{3} f(x)\sin\left(\frac{n\pi x}{3}\right)dx = \frac{1}{3}\int_{0}^{3} x\sin\left(\frac{n\pi x}{3}\right)dx$$

$$= -\frac{3}{n\pi}\cos(n\pi)$$

Again, we are careful to split up the integral because the integration takes place over two domains with separate functions. Though for [−3, 0], $f(x)=0$, it's safe to write out the first integral with the limits over the whole domain, that is [−3, 3].

Since $\cos(n\pi) = (-1)^n$, the Fourier series of *f* on [−3, 3] is

$$\frac{3}{4}+\sum_{n=1}^{\infty}\left[\frac{3}{n^2\pi^2}\left[(-1)^n-1\right]\cos\left(\frac{n\pi x}{3}\right)-\frac{3}{n\pi}(-1)^n\sin\left(\frac{n\pi x}{3}\right)\right]$$

Our equations are getting rather lengthy. Get used to it. There is no compromise when writing Fourier series and sometimes they do get rather long.