

AMC

Functions 1993 AHSME #12

Suppose that for all $x > 0$ we have $f(2x) = \frac{2}{2+x}$. What is $2f(x)$?

Solution

Here is an easy problem to get ourselves acquainted with functions. We shall solve it using a systemic approach and then when we're familiar with it, we'll use a short cut.

This is essentially a type of question which requires a 'change in variable'. What we need is to explicitly find $f(x)$ from a given $f(_)$ where the blank can be any sort of expression written in x . In this case, the blank is $2x$, fairly easy to manipulate but we'll go through the full steps. Lets a temporary variable be y such that

$$y = 2x$$
$$\text{and } x = y/2$$

Substituting this into the function we have

$$f(y) = \frac{2}{2+y/2} = \frac{4}{4+y}$$

This function is written in terms of y . We can simply change it back to x because what is written on the left hand side and right hand side is BOTH in terms of y . This gives us

$$f(x) = \frac{4}{4+x}$$

And so

$$2f(x) = 2 \cdot \frac{4}{4+x} = \frac{8}{4+x}$$

Problem solved. Now for the short cut method. From the $f(2x)$, we try to find an expression such that when we substitute into the x , we get $f(x)$. This isn't too difficult to find. We 'substitute' $x/2$ into x . While this may not be too algebraically correct, it still works. The more correct way is to substitute a dummy variable of say y but I assume the reader knows what I mean. Doing likewise and multiplying by 2, we have

$$2 \cdot f\left(2\frac{x}{2}\right) = 2f(x) = 2 \cdot \frac{2}{2 + x/2} = \frac{2}{4 + x/2} = \frac{4}{4 + x}$$

as intended.

This is the short cut in solving this question. However, take note that sometimes functions given in the AHSME aren't too friendly like in terms of $2x$ and so the proper method of the change in variable is desired. There is a question which calls such in my site. Hope you look at it.