Vector Integral Calculus

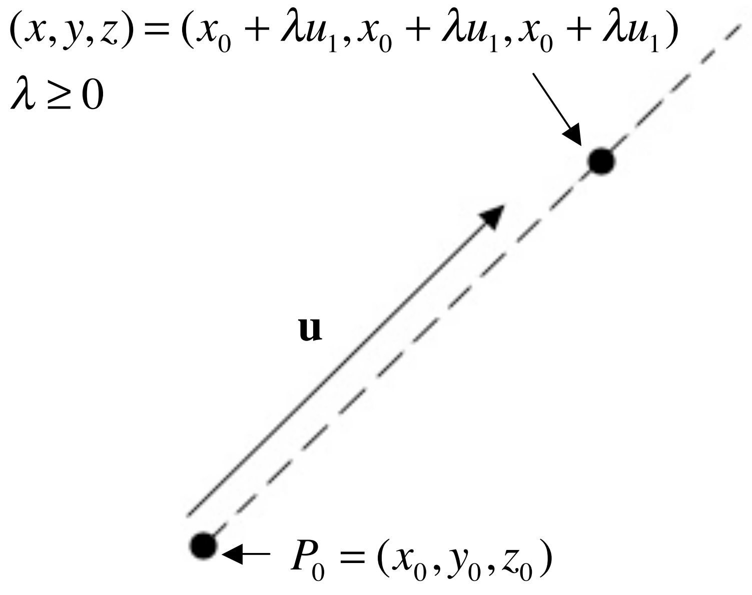
Defining the Directional Derivative

After talking about the gradient vector field, we did mention that the gradient is related to the directional derivative of φ. In this lesson and the next, we shall make the relationship clear.

Let P_0 have the coordinates (x_0, y_0, z_0) and let (x, y, z) be any point on the line from P_0 in the direction of the vector $\mathbf{u}$. Using normal vector algebra, we can specify the vector

$$(x - x_0)\mathbf{i} + (y - y_0)\mathbf{j} + (z - z_0)\mathbf{k}$$

which is parallel and in the same direction as $\mathbf{u}$ as shown below.



Hence for some $\lambda > 0$, we write

$$(x - x_0)\mathbf{i} + (y - y_0)\mathbf{j} + (z - z_0)\mathbf{k} = \lambda\mathbf{u}$$

We define $\mathbf{u}$ as $\lambda\mathbf{u} = \lambda u_1\mathbf{i} + \lambda u_2\mathbf{j} + \lambda u_3\mathbf{k}$ and then equate the separate unit vectors to give

$$x - x_0 = \lambda u_1, \quad y - y_0 = \lambda u_2, \quad z - z_0 = \lambda u_3$$

and after rearranging,

$$x = x_0 + \lambda u_1, \quad y = y_0 + \lambda u_2, \quad z = z_0 + \lambda u_3$$

This set of equations giving us the coordinates of (x, y, z) is key in finding the gradient of φ. They link a certain point P_0 and the unit vector in which we are travelling to by giving the coordinates (x, y, z) in a 'general form'. Thus, we apply the function φ.

$$\varphi(x, y, z) = \varphi(x_0 + \lambda u_1, y_0 + \lambda u_2, z_0 + \lambda u_3)$$

Finally, we can find the derivative of φ by differentiating with respect to the constant λ. All the definitions of the directional derivative is taken into account here – The unit vector $\mathbf{u}$ is represented by u_1, u_2, u_3 and the point P_0 is represented by x_0, y_0, z_0. The rate of change is thus to differentiate w.r.t to λ.

We shall continue this in our next lesson and finally see how the del operator is used in finding the directional derivative.