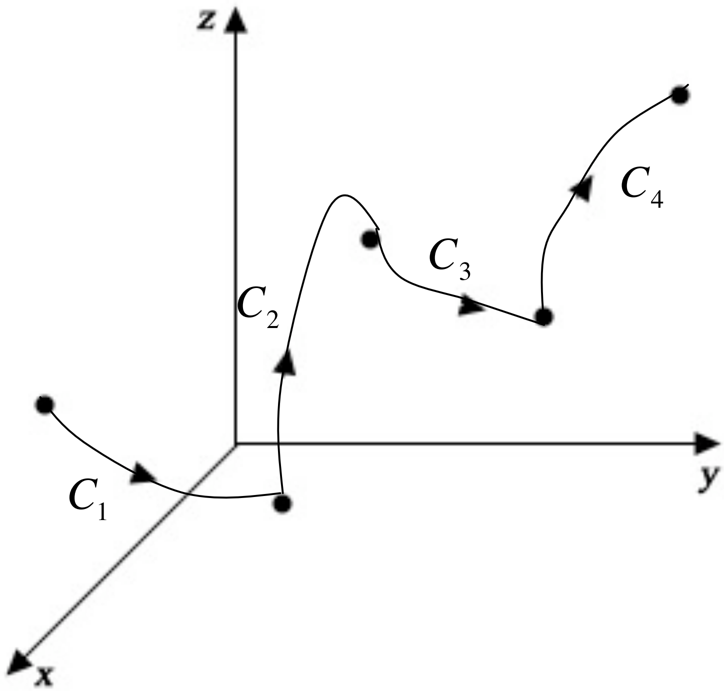
Vector Integral Calculus

Line Integral of a Piecewise smooth curve

This is a short lesson on how the line integral is applied to a piecewise smooth curve. A typical piecewise smooth curve is one where there are points at which the curve as no tangent.

Formally stated, a curve *C* having continuous position vector $\vec{\mathbf{R}}(t)$ is *piecewise smooth* if $\vec{\mathbf{R}}'(t)$ is continuous and different from $\mathbf{O}$ at all but possibly a finite number of values of *t*.

One such example is shown below.



Notice that there are points on the curve, namely the dots, at which there is no tangent vector.

To evaluate such a line integral, the rule we use is quite simple. If *C* is piecewise smooth, consisting of smooth curves $C_1, C_2, \ldots, C_n$, the line integral of $\vec{\mathbf{F}}$ over *C* is defined to be the sum of the line integrals of $\vec{\mathbf{F}}$ over each of the smooth curves making up *C* or written symbolically as

$$\int_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{R}} = \int_{C_1} \vec{\mathbf{F}} \bullet d\vec{\mathbf{R}} + \int_{C_2} \vec{\mathbf{F}} \bullet d\vec{\mathbf{R}} + \cdots + \int_{C_n} \vec{\mathbf{F}} \bullet d\vec{\mathbf{R}}$$

There are a few things to keep track of.

1. The orientation along C must be maintained over the curves $C_1, C_2, \ldots, C_n$ implying that the initial point of C_i is the terminal point of the smooth curve preceding it namely C_{i-1}. Another way of looking at it is that the smooth curves must all be connected.

2. The vector field $\vec{\mathbf{F}}$ is the *same* for all the individual smooth curves but the position vector $\vec{\mathbf{R}}$ is *different*. It is the position vector of that specific curve in the subscript of the integral sign, i.e., $\vec{\mathbf{R}}$ in the integral $\int_{C_2}$ must be the position vector describing C_2.

3. The usual rules of calculating line integrals apply for each individual smooth curve.