Time Travel
Mathematical Wormhole

Any science fiction fan, or movie buff for this matter, will be familiar with the concept of hyperspace wormholes. Watch any *Star Trek* movie and you'll see the Enterprise zip through two points in space faster than the time takes to conventionally travel in a straight line. The possibility of wormholes elevated to another level when Einstein published his general theory of relativity. In this section, we'll see that mathematicians had this idea in mind even before the physics community.

In high school calculus course, you'll come across the differential arc length ds along the curve y given by

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{(dx)^2}\sqrt{\frac{1}{(dx)^2}\left[(dx)^2 + (dy)^2\right]}$$

$$= dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

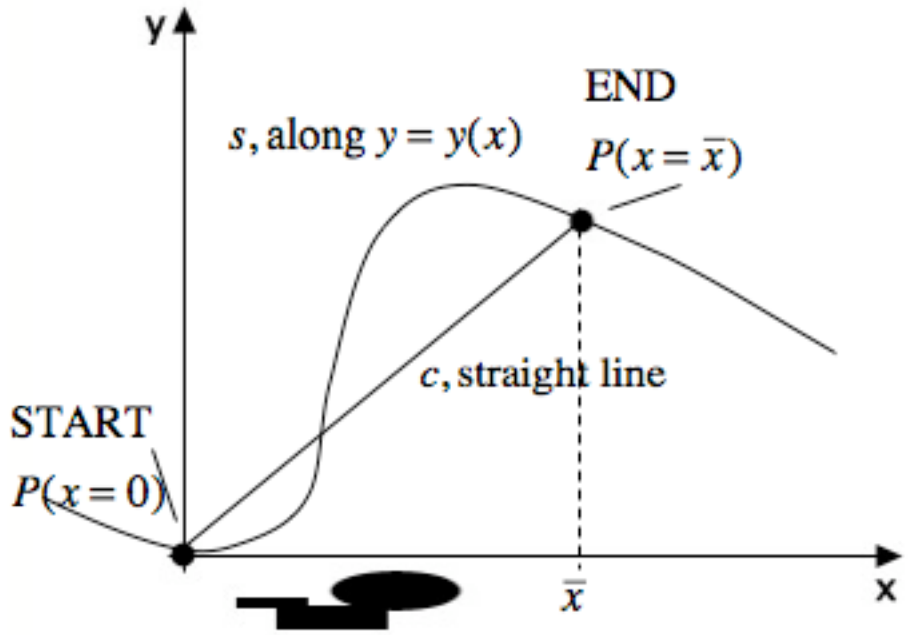
This isn't too hard to derive. Simply notice that ds, dx and dy forms a right-angle for small values and then apply Pythagoreans theorem.

The arc length, *s*, from $x = 0$ to $x = \bar{x}$ is simply

$$s = \int_0^{\bar{x}} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

Here is where our mathematician hero comes in. American mathematician Edward Kasner in 1914 showed how if the function $y(x)$ is complex valued, we have just created wormhole in space!

Let's set out the situation formally. We are all riding in the Enterprise and need to travel from point *P* at $x = 0$ in space to point *Q* at $x = \bar{x}$ along distance *s*. Our route of travel is marked out by the graph $y = y(x)$ as shown below.



Alternatively, we could take what everyone calls the shortest distance between two points, that is the chord between *P* and *Q* and travel through the distance *c*. Clearly, we all know that $c < s$. Upon closer inspection, we can also write

$$\lim_{\bar{x}\to 0}\frac{s}{c}=1$$

which should be no surprise as the two points gets closer, they merge into each other and so their ratio is 1. Or is it?

Well, it certainly is for real-valued functions. Let's just take a tour with a complex function. While complex-valued functions can't be drawn on a curve – draw for me $y(x)=x^2+ix$ and I'll give you a hundred bucks – that does not stop us from performing the usual algebraic manipulation. Please read carefully that we are using the function $y(x)=x^2+ix$ and not $z=a+bi$ which can be drawn. This time, its impossible to have the value x^2+ix on the *y* axis.