

AMC

Polynomials and their Zeros 1999 AHSME #17

Let $P(x)$ be a polynomial which when divided by $x - 19$ has the remainder 99, and when divided by $x - 99$ has remainder 19. What is the remainder when $P(x)$ is divided by $(x - 19)(x - 99)$?

Solution

This question can look intimidating on first sight but after some careful implementation of the various theorems, the answer pretty much falls in place.

We notice that $(x - 19)(x - 99)$ is a quadratic polynomial. The remainder when this is divided into $P(x)$ will be linear, that is,

$$P(x) = (x - 19)(x - 99)Q(x) + ax + b$$

for some constants a and b .

If we were to put $x = 99$ and $x = 19$, the linear factor theorem implies that

$$99 = P(19) = 19a + b$$

$$19 = P(99) = 99a + b$$

Subtracting these equations and substituting gives

$$80a = -80$$

So $a = -1$ and $b = 99 - (-1)19 = 118$. The remainder is therefore $-x + 118$.